<u>EJERCICIO 1 (45:13)</u>

Dados:

$$\underline{\epsilon}_0 = \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix} \qquad \underline{\epsilon}_1 = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ \cos\theta\cos\phi - i\sin\phi \\ \cos\theta\sin\phi + i\cos\phi \\ -\sin\theta \end{pmatrix} \quad \underline{\epsilon}_2 = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ \cos\theta\cos\phi + i\sin\phi \\ \cos\theta\sin\phi - i\cos\phi \\ -\sin\theta \end{pmatrix}$$

$$\underline{\epsilon}_3 = \begin{pmatrix} 0 \\ \sin\theta\cos\phi \\ \sin\theta\sin\phi \\ \cos\theta \end{pmatrix}$$

Comprobar que $\underline{\epsilon}_r{}^* \cdot \underline{\epsilon}_s = g_{rs}$

Recordar que $\underline{\epsilon}_r{}^* \cdot \underline{\epsilon}_s = \underline{\epsilon}_{r_\mu}{}^* \, g^{\mu\nu} \, \underline{\epsilon}_{s_\nu}$

Con la métrica $g^{\mu\nu} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$

$$\underline{\epsilon}_0{}^* \, \underline{\epsilon}_0 = \underline{\epsilon}_0{}^\dagger \, g^{\mu\nu} \, \underline{\epsilon}_0 = (1 \quad 0 \quad 0 \quad 0)\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}\begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix} = (1 \quad 0 \quad 0 \quad 0)\begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix} = 1 + 0 + 0 + 0$$

$$\boxed{\underline{\epsilon}_0{}^* \, \underline{\epsilon}_0 = 1 = g_{00}}$$

$$\underline{\epsilon}_1{}^* \, \underline{\epsilon}_1 = \underline{\epsilon}_1{}^\dagger \, g^{\mu\nu} \, \underline{\epsilon}_1 =$$

$$= \frac{1}{\sqrt{2}}(0 \quad \cos\theta\cos\phi + i\sin\phi \quad \cos\theta\sin\phi - i\cos\phi \quad -\sin\theta)\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}\frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ \cos\theta\cos\phi - i\sin\phi \\ \cos\theta\sin\phi + i\cos\phi \\ -\sin\theta \end{pmatrix}$$

$$\underline{\epsilon}_1{}^* \, \underline{\epsilon}_1 = \frac{1}{2}(0 \quad \cos\theta\cos\phi + i\sin\phi \quad \cos\theta\sin\phi - i\cos\phi \quad -\sin\theta)\begin{pmatrix} 0 \\ -\cos\theta\cos\phi + i\sin\phi \\ -\cos\theta\sin\phi - i\cos\phi \\ \sin\theta \end{pmatrix}$$

$$\underline{\epsilon}_1{}^* \, \underline{\epsilon}_1 = \frac{1}{2}(0 + (\cos\theta\cos\phi + i\sin\phi)(-\cos\theta\cos\phi + i\sin\phi) + (\cos\theta\sin\phi - i\cos\phi)(-\cos\theta\sin\phi - i\cos\phi) + (-\sin\theta)\sin\theta)$$

$$\underline{\epsilon}_1{}^* \, \underline{\epsilon}_1 = \frac{1}{2}\{0 - ((\cos\theta\cos\phi)^2 - i^2(\sin\phi)^2) - ((\cos\theta\sin\phi)^2 - i^2(\cos\phi)^2) - (\sin\theta)^2\}$$

$$\underline{\epsilon}_1{}^* \, \underline{\epsilon}_1 = \frac{1}{2}\{-(\cos\theta\cos\phi)^2 - (\sin\phi)^2 - (\cos\theta\sin\phi)^2 - (\cos\phi)^2 - (\sin\theta)^2\}$$

$$\underline{\epsilon_1}^* \, \underline{\epsilon_1} = \frac{1}{2}\{-1 - (\cos\theta)^2(\cos\phi)^2 - (\cos\theta)^2(\sin\phi)^2 - (\sin\theta)^2\}$$

$$\underline{\epsilon_1}^* \, \underline{\epsilon_1} = \frac{1}{2}\{-1 - (\cos\theta)^2((\cos\phi)^2 + (\sin\phi)^2) - (\sin\theta)^2\}$$

$$\underline{\epsilon_1}^* \, \underline{\epsilon_1} = \frac{1}{2}\{-1 - (\cos\theta)^2 \, 1 - (\sin\theta)^2\}$$

$$\underline{\epsilon_1}^* \, \underline{\epsilon_1} = \frac{1}{2}\{-1 - ((\cos\theta)^2 + (\sin\theta)^2)\} = \frac{1}{2}\{-1 - 1\}$$

$$\boxed{\boldsymbol{\underline{\epsilon_1}^* \, \underline{\epsilon_1} = -1 = g_{11}}}$$

$$\underline{\epsilon_2}^* \, \underline{\epsilon_2} = \underline{\epsilon_2}^\dagger \, g^{\mu\nu} \, \underline{\epsilon_2} =$$

$$= \frac{1}{\sqrt{2}}\begin{pmatrix}0 & \cos\theta\cos\phi - i\sin\phi & \cos\theta\sin\phi + i\cos\phi & -\sin\theta\end{pmatrix}\begin{pmatrix}1 & 0 & 0 & 0\\ 0 & -1 & 0 & 0\\ 0 & 0 & -1 & 0\\ 0 & 0 & 0 & -1\end{pmatrix}\frac{1}{\sqrt{2}}\begin{pmatrix}0\\ \cos\theta\cos\phi + i\sin\phi\\ \cos\theta\sin\phi - i\cos\phi\\ -\sin\theta\end{pmatrix}$$

$$\underline{\epsilon_2}^* \, \underline{\epsilon_2} = \frac{1}{2}\begin{pmatrix}0 & \cos\theta\cos\phi - i\sin\phi & \cos\theta\sin\phi + i\cos\phi & -\sin\theta\end{pmatrix}\begin{pmatrix}0\\ -\cos\theta\cos\phi - i\sin\phi\\ -\cos\theta\sin\phi + i\cos\phi\\ \sin\theta\end{pmatrix}$$

$$\underline{\epsilon_2}^* \, \underline{\epsilon_2} = \frac{1}{2}(0 + (\cos\theta\cos\phi - i\sin\phi)(-\cos\theta\cos\phi - i\sin\phi) + (\cos\theta\sin\phi + i\cos\phi)(-\cos\theta\sin\phi + i\cos\phi) + (-\sin\theta)\sin\theta)$$

$$\underline{\epsilon_2}^* \, \underline{\epsilon_2} = \frac{1}{2}\{0 - ((\cos\theta\cos\phi)^2 - i^2(\sin\phi)^2) - ((\cos\theta\sin\phi)^2 - i^2(\cos\phi)^2) - (\sin\theta)^2\}$$

$$\underline{\epsilon_2}^* \, \underline{\epsilon_2} = \frac{1}{2}\{-(\cos\theta\cos\phi)^2 - (\sin\phi)^2 - (\cos\theta\sin\phi)^2 - (\cos\phi)^2 - (\sin\theta)^2\}$$

$$\underline{\epsilon_2}^* \, \underline{\epsilon_2} = \frac{1}{2}\{-1 - (\cos\theta)^2(\cos\phi)^2 - (\cos\theta)^2(\sin\phi)^2 - (\sin\theta)^2\}$$

$$\underline{\epsilon_2}^* \, \underline{\epsilon_2} = \frac{1}{2}\{-1 - (\cos\theta)^2((\cos\phi)^2 + (\sin\phi)^2) - (\sin\theta)^2\}$$

$$\underline{\epsilon_2}^* \, \underline{\epsilon_2} = \frac{1}{2}\{-1 - (\cos\theta)^2 \, 1 - (\sin\theta)^2\}$$

$$\underline{\epsilon_2}^* \, \underline{\epsilon_2} = \frac{1}{2}\{-1 - ((\cos\theta)^2 + (\sin\theta)^2)\} = \frac{1}{2}\{-1 - 1\}$$

$$\boxed{\boldsymbol{\underline{\epsilon_2}^* \, \underline{\epsilon_2} = -1 = g_{22}}}$$

$$\underline{\epsilon_3}^* \, \underline{\epsilon_3} = \underline{\epsilon_3}^\dagger \, g^{\mu\nu} \, \underline{\epsilon_3} = \begin{pmatrix}0 & \sin\theta\cos\phi & \sin\theta\sin\phi & \cos\theta\end{pmatrix}\begin{pmatrix}1 & 0 & 0 & 0\\ 0 & -1 & 0 & 0\\ 0 & 0 & -1 & 0\\ 0 & 0 & 0 & -1\end{pmatrix}\begin{pmatrix}0\\ \sin\theta\cos\phi\\ \sin\theta\sin\phi\\ \cos\theta\end{pmatrix}$$

$$\underline{\epsilon_3}^* \, \underline{\epsilon_3} = \underline{\epsilon_3}^\dagger \, g^{\mu\nu} \, \underline{\epsilon_3} = (0 \quad \sin\theta\cos\phi \quad \sin\theta\sin\phi \quad \cos\theta)\begin{pmatrix} 0 \\ -\sin\theta\cos\phi \\ -\sin\theta\sin\phi \\ -\cos\theta \end{pmatrix}$$

$$\underline{\epsilon_3}^* \, \underline{\epsilon_3} = (0 - \sin\theta\cos\phi\sin\theta\cos\phi - \sin\theta\sin\phi\sin\theta\sin\phi - \cos\theta\cos\theta)$$

$$\underline{\epsilon_3}^* \, \underline{\epsilon_3} = \{0 - (\sin\theta)^2(\cos\phi)^2 - (\sin\theta)^2(\sin\phi)^2 - (\cos\theta)^2\}$$

$$\underline{\epsilon_3}^* \, \underline{\epsilon_3} = \{-(\sin\theta)^2((\cos\phi)^2 + (\sin\phi)^2) - (\cos\theta)^2\}$$

$$\underline{\epsilon_3}^* \, \underline{\epsilon_3} = \{-(\sin\theta)^2 - (\cos\theta)^2\}$$

$$\boxed{\underline{\boldsymbol{\epsilon_3}}^* \, \underline{\boldsymbol{\epsilon_3}} = \mathbf{-1} = \boldsymbol{g_{33}}}$$

Calculamos el resto de los términos, recordando que se trata de una base, es decir, cuadrivectores linealmente independientes, por lo que los valores de aquéllos deben ser nulos.

Como $\underline{\epsilon_0} = \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix}$ y las componentes temporales de los otros cuadrivectores son nulas, y considerando la conmutatividad del producto escalar, resulta que:

$$\boxed{\underline{\boldsymbol{\epsilon_0}}^* \cdot \underline{\boldsymbol{\epsilon_1}} = \boldsymbol{g_{01}} = \boldsymbol{g_{10}} = \mathbf{0}}$$

$$\boxed{\underline{\boldsymbol{\epsilon_0}}^* \cdot \underline{\boldsymbol{\epsilon_2}} = \boldsymbol{g_{02}} = \boldsymbol{g_{20}} = \mathbf{0}}$$

$$\boxed{\underline{\boldsymbol{\epsilon_0}}^* \cdot \underline{\boldsymbol{\epsilon_3}} = \boldsymbol{g_{03}} = \boldsymbol{g_{30}} = \mathbf{0}}$$

$$\underline{\epsilon_1}^* \, \underline{\epsilon_2} = \underline{\epsilon_1}^\dagger \, g^{\mu\nu} \, \underline{\epsilon_2}$$

$$= \frac{1}{\sqrt{2}}(0 \quad \cos\theta\cos\phi + i\sin\phi \quad \cos\theta\sin\phi - i\cos\phi \quad -\sin\theta)\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}\frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ \cos\theta\cos\phi + i\sin\phi \\ \cos\theta\sin\phi - i\cos\phi \\ -\sin\theta \end{pmatrix}$$

$$\underline{\epsilon_1}^* \, \underline{\epsilon_2} = \frac{1}{2}(0 \quad \cos\theta\cos\phi + i\sin\phi \quad \cos\theta\sin\phi - i\cos\phi \quad -\sin\theta)\begin{pmatrix} 0 \\ -\cos\theta\cos\phi - i\sin\phi \\ -\cos\theta\sin\phi + i\cos\phi \\ \sin\theta \end{pmatrix}$$

$$\underline{\epsilon_1}^* \, \underline{\epsilon_2} = \frac{1}{2}(0 + (\cos\theta\cos\phi + i\sin\phi)(-\cos\theta\cos\phi - i\sin\phi) + (\cos\theta\sin\phi - i\cos\phi)(-\cos\theta\sin\phi + i\cos\phi) + (-\sin\theta)\sin\theta)$$

$$\underline{\epsilon_1}^* \, \underline{\epsilon_2} = \frac{1}{2}(0 - (\cos\theta\cos\phi + i\sin\phi)^2 - (\cos\theta\sin\phi - i\cos\phi)^2 - (\sin\theta)^2)$$

$$\underline{\epsilon_1}^* \, \underline{\epsilon_2} = \frac{1}{2}(-(\cos\theta\cos\phi)^2 - 2\cos\theta\cos\phi\, i\sin\phi + (\sin\phi)^2 - (\cos\theta\sin\phi)^2 + 2\cos\theta\sin\phi\, i\cos\phi + (\cos\phi)^2 - (\sin\theta)^2)$$

$$\underline{\epsilon_1}^* \underline{\epsilon_2} = \frac{1}{2}(-(\cos\theta)^2((\cos\phi)^2 + (\sin\phi)^2) - 0 + 1 - (\sin\theta)^2)$$

$$\underline{\epsilon_1}^* \underline{\epsilon_2} = \frac{1}{2}(-(\cos\theta)^2((\cos\phi)^2 + (\sin\phi)^2) + 1 - (\sin\theta)^2)$$

$$\underline{\epsilon_1}^* \underline{\epsilon_2} = \frac{1}{2}(-(\cos\theta)^2 + 1 - (\sin\theta)^2)$$

$$\underline{\epsilon_1}^* \underline{\epsilon_2} = \frac{1}{2}(-1 + 1)$$

$$\boxed{\underline{\boldsymbol{\epsilon_1}}^* \underline{\boldsymbol{\epsilon_2}} = \boldsymbol{g_{12}} = \boldsymbol{g_{21}} = \boldsymbol{0}}$$

$$\underline{\epsilon_1}^* \underline{\epsilon_3} = \underline{\epsilon_1}^\dagger g^{\mu\nu} \underline{\epsilon_3}$$

$$= \frac{1}{\sqrt{2}}\begin{pmatrix}0 & \cos\theta\cos\phi + i\sin\phi & \cos\theta\sin\phi - i\cos\phi & -\sin\theta\end{pmatrix}\begin{pmatrix}1 & 0 & 0 & 0\\ 0 & -1 & 0 & 0\\ 0 & 0 & -1 & 0\\ 0 & 0 & 0 & -1\end{pmatrix}\begin{pmatrix}0\\ \sin\theta\cos\phi\\ \sin\theta\sin\phi\\ \cos\theta\end{pmatrix}$$

$$\underline{\epsilon_1}^* \underline{\epsilon_3} = \frac{1}{\sqrt{2}}\begin{pmatrix}0 & \cos\theta\cos\phi + i\sin\phi & \cos\theta\sin\phi - i\cos\phi & -\sin\theta\end{pmatrix}\begin{pmatrix}0\\ -\sin\theta\cos\phi\\ -\sin\theta\sin\phi\\ -\cos\theta\end{pmatrix}$$

$$\underline{\epsilon_1}^* \underline{\epsilon_3} = \frac{1}{\sqrt{2}}(0 + (\cos\theta\cos\phi + i\sin\phi)(-\sin\theta\cos\phi) + (\cos\theta\sin\phi - i\cos\phi)(-\sin\theta\sin\phi) + \sin\theta\cos\theta)$$

$$\underline{\epsilon_1}^* \underline{\epsilon_3} = \frac{1}{\sqrt{2}}(-\cos\theta\cos\phi\sin\theta\cos\phi - i\sin\phi\sin\theta\cos\phi - \cos\theta\sin\phi\sin\theta\sin\phi + i\cos\phi\sin\theta\sin\phi + \sin\theta\cos\theta)$$

$$\underline{\epsilon_1}^* \underline{\epsilon_3} = \frac{1}{\sqrt{2}}(-\cos\theta((\cos\phi)^2 - (\sin\phi)^2)\sin\theta + 0 + \sin\theta\cos\theta)$$

$$\underline{\epsilon_1}^* \underline{\epsilon_3} = \frac{1}{\sqrt{2}}(-\cos\theta\sin\theta + \sin\theta\cos\theta)$$

$$\boxed{\underline{\boldsymbol{\epsilon_1}}^* \underline{\boldsymbol{\epsilon_3}} = \boldsymbol{g_{13}} = \boldsymbol{g_{31}} = \boldsymbol{0}}$$

$$\underline{\epsilon_2}^* \underline{\epsilon_3} = \underline{\epsilon_2}^\dagger g^{\mu\nu} \underline{\epsilon_3}$$

$$= \frac{1}{\sqrt{2}}\begin{pmatrix}0 & \cos\theta\cos\phi - i\sin\phi & \cos\theta\sin\phi + i\cos\phi & -\sin\theta\end{pmatrix}\begin{pmatrix}1 & 0 & 0 & 0\\ 0 & -1 & 0 & 0\\ 0 & 0 & -1 & 0\\ 0 & 0 & 0 & -1\end{pmatrix}\begin{pmatrix}0\\ \sin\theta\cos\phi\\ \sin\theta\sin\phi\\ \cos\theta\end{pmatrix}$$

$$\underline{\epsilon_2}^* \underline{\epsilon_3} = \frac{1}{\sqrt{2}}\begin{pmatrix}0 & \cos\theta\cos\phi - i\sin\phi & \cos\theta\sin\phi + i\cos\phi & -\sin\theta\end{pmatrix}\begin{pmatrix}0\\ -\sin\theta\cos\phi\\ -\sin\theta\sin\phi\\ -\cos\theta\end{pmatrix}$$

$$\underline{\epsilon_2}^* \underline{\epsilon_3} = \frac{1}{\sqrt{2}}(0 + (\cos\theta\cos\phi - i\sin\phi)(-\sin\theta\cos\phi) + (\cos\theta\sin\phi + i\cos\phi)(-\sin\theta\sin\phi) + \sin\theta\cos\theta)$$

$$\underline{\epsilon_2}^* \underline{\epsilon_3} = \frac{1}{\sqrt{2}}(-\cos\theta\cos\phi\sin\theta\cos\phi + i\sin\phi\sin\theta\cos\phi - \cos\theta\sin\phi\sin\theta\sin\phi - i\cos\phi\sin\theta\sin\phi + \sin\theta\cos\theta)$$

$$\underline{\epsilon_2}^* \underline{\epsilon_3} = \frac{1}{\sqrt{2}}(-\cos\theta\sin\theta\,((\cos\phi)^2 + (\sin\phi)^2) + 0 + \sin\theta\cos\theta)$$

$$\underline{\epsilon_2}^* \underline{\epsilon_3} = \frac{1}{\sqrt{2}}(-\cos\theta\sin\theta + \sin\theta\cos\theta)$$

$$\boxed{\underline{\boldsymbol{\epsilon_2}}^* \underline{\boldsymbol{\epsilon_3}} = \boldsymbol{g_{23}} = \boldsymbol{g_{32}} = \boldsymbol{0}}$$

<u>EJERCICIO 2 (48:21)</u>

Verificar que la suma de los campos eléctrico y magnético de las polarizaciones correspondientes a $\underline{\epsilon_0}$ y a $\underline{\epsilon_3}$ dan cero.

Esta verificación fue realizada por Javier en el capítulo 13 (1:56) empleando la fórmula del capítulo 65 del curso de QFT: $\vec{E} = \left(b^0\vec{k} - w\vec{b}\right)\cos(k\cdot x)$

Vamos a trabajar con las fórmulas 62.1 del formulario del curso de QFT (Crul et al.), siendo:

$\underline{A} = \underline{\epsilon_\mu}\sin(p\cdot x)$

<u>Campo Eléctrico:</u> $\boldsymbol{E^i = -\partial_i A^0 - \partial_0 A^i}$

Para $\underline{\epsilon_0} = \begin{pmatrix}1\\0\\0\\0\end{pmatrix}$

$A^0 = 1\sin(p\cdot x)$

$A^i = 0$

$E^i{}_{\underline{\epsilon_0}} = -\partial_i A^0 = -\partial_i(p\cdot x)\cos(p\cdot x) = -\partial_i\left(E_p x_0 - p^i x_i\right)\cos(p\cdot x) = -\left(-p^i\right)\cos(p\cdot x)$

$E^i{}_{\underline{\epsilon_0}} = p^i\cos(p\cdot x)$

Como:

$\vec{p} = |p|\begin{pmatrix}\sin\theta\cos\phi\\ \sin\theta\sin\phi\\ \cos\theta\end{pmatrix}$

Resulta:

$$\vec{E}_{\underline{\epsilon}_0} = |p| \begin{pmatrix} \sin\theta\cos\phi \\ \sin\theta\sin\phi \\ \cos\theta \end{pmatrix} \cos(p\cdot x)$$

Para $\underline{\epsilon}_3 = \begin{pmatrix} 0 \\ \sin\theta\cos\phi \\ \sin\theta\sin\phi \\ \cos\theta \end{pmatrix}$

$A^0 = 0$

$A^i = \underline{\epsilon}_3{}^i \sin(p\cdot x)$

$E^i{}_{\underline{\epsilon}_3} = -\partial_0 A^i = -\underline{\epsilon}_3{}^i \partial_0(p\cdot x)\cos(p\cdot x) = -\underline{\epsilon}_3{}^i \partial_0\left(E_p x_0 - p^i x_i\right)\cos(p\cdot x) = -\underline{\epsilon}_3{}^i E_p \cos(p\cdot x)$

Resulta:

$$\vec{E}_{\underline{\epsilon}_3} = -E_p \begin{pmatrix} \sin\theta\cos\phi \\ \sin\theta\sin\phi \\ \cos\theta \end{pmatrix} \cos(p\cdot x)$$

Para los fotones se cumple que $E_p = |p|$, de donde se concluye que:

$$\boxed{\vec{\boldsymbol{E}}_{\underline{\epsilon}_0} = -\vec{\boldsymbol{E}}_{\underline{\epsilon}_3}}$$

<u>**Campo Magnético:**</u> $\boldsymbol{B^i = \varepsilon_{ijk}\, \partial_j A^k}$

Considerando que:

$\partial_i A^j = \underline{\epsilon}_3{}^j \partial_i(p\cdot x)\cos(p\cdot x) = -\underline{\epsilon}_3{}^j p^i \cos(p\cdot x)$

$\partial_1 A^2 = -\underline{\epsilon}_3{}^2 p^1 \cos(p\cdot x)$

$\partial_2 A^1 = -\underline{\epsilon}_3{}^1 p^2 \cos(p\cdot x)$

$\partial_1 A^3 = -\underline{\epsilon}_3{}^3 p^1 \cos(p\cdot x)$

$\partial_3 A^1 = -\underline{\epsilon}_3{}^1 p^3 \cos(p\cdot x)$

$\partial_2 A^3 = -\underline{\epsilon}_3{}^3 p^2 \cos(p\cdot x)$

$\partial_3 A^2 = -\underline{\epsilon}_3{}^2 p^3 \cos(p\cdot x)$

Para $\underline{\epsilon}_0 = \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix}$

Como las componentes espaciales son nulas, resulta inmediatamente que:

$$\vec{B}_{\underline{\epsilon}_0} = 0$$

Para $\underline{\epsilon}_3 = \begin{pmatrix} 0 \\ \sin\theta\cos\phi \\ \sin\theta\sin\phi \\ \cos\theta \end{pmatrix}$

$B^1{}_{\underline{\epsilon}_3} = \varepsilon_{123}\partial_2 A^3 + \varepsilon_{132}\partial_3 A^2 = \partial_2 A^3 - \partial_3 A^2 = \left(-\underline{\epsilon}_3{}^3 p^2 + \underline{\epsilon}_3{}^2 p^3\right)\cos(p \cdot x)$

$B^1{}_{\underline{\epsilon}_3} = (-\cos\theta \sin\theta \sin\phi + \sin\theta \sin\phi \cos\theta)\cos(p \cdot x)$

$B^1{}_{\underline{\epsilon}_3} = 0$

$B^2{}_{\underline{\epsilon}_3} = \varepsilon_{213}\partial_1 A^3 + \varepsilon_{231}\partial_3 A^1 = -\partial_1 A^3 + \partial_3 A^1 = \left(\underline{\epsilon}_3{}^3 p^1 - \underline{\epsilon}_3{}^1 p^3\right)\cos(p \cdot x)$

$B^2{}_{\underline{\epsilon}_3} = (\cos\theta \sin\theta \cos\phi - \sin\theta \cos\phi \cos\theta)\cos(p \cdot x)$

$B^2{}_{\underline{\epsilon}_3} = 0$

$B^3{}_{\underline{\epsilon}_3} = \varepsilon_{312}\partial_1 A^2 + \varepsilon_{321}\partial_2 A^1 = \partial_1 A^2 - \partial_2 A^1 = \left(-\underline{\epsilon}_3{}^2 p^1 + \underline{\epsilon}_3{}^1 p^2\right)\cos(p \cdot x)$

$B^3{}_{\underline{\epsilon}_3} = (-\sin\theta \sin\phi \sin\theta \cos\phi + \sin\theta \cos\phi \sin\theta \sin\phi)\cos(p \cdot x)$

$B^3{}_{\underline{\epsilon}_3} = 0$

$$\vec{B}_{\underline{\epsilon}_3} = 0$$

$$\boxed{\vec{\boldsymbol{B}}_{\underline{\epsilon}_0} = \vec{\boldsymbol{B}}_{\underline{\epsilon}_3} = \mathbf{0}}$$